

module 3

1. Find the trigonometrical, exponential and polar forms of the vector $8 + j6$.

$$r = \sqrt{8^2 + 6^2} = 10$$

$$\phi = \tan^{-1} \frac{6}{8} = \underline{36.86}$$

trigonometric form - $10 (\cos 36.86 + j \sin 36.86)$

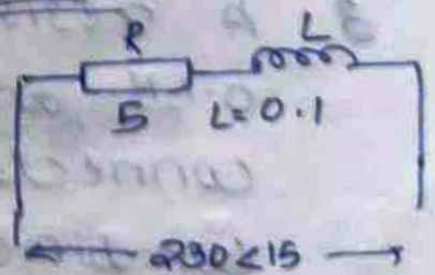
exponential - $10 e^{j36.86}$

polar - $10 \angle 36.86$

2. A sinusoidal voltage of $V = 230 \angle 15$ of freq. 50 Hz is applied to series RL circuit consisting of $R = 5 \Omega$ and $L = 0.1 \text{ H}$. calculate i) rms value of current and its phase angle. ii) power factor iii) avg. power iv) reactive power and v) apparent power drawn by the ckt.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{5^2 + 31.41^2} = 31.6$$

$$i) I = \frac{V}{Z} = \frac{230 \angle 15}{5 + j31.41} \text{ A}$$



$$X_L = \omega L = 2\pi fL$$

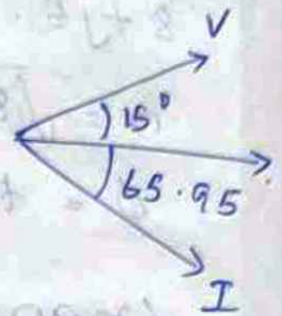
$$= 2 \times \pi \times 50 \times 0.1 = 31.41 \Omega$$

$$Z = (5 + j31.41) \Omega$$

$$I = \frac{V}{Z} = \frac{230 \angle 15^\circ}{5 + j31.41} = \underline{\underline{7.23 \angle -65.95^\circ \text{ A}}}$$

$$\theta = 65.95 + 15 = 80.95^\circ$$

$$p.f. = \cos \theta = \cos 80.95^\circ = \underline{\underline{0.157 \text{ lag}}}$$



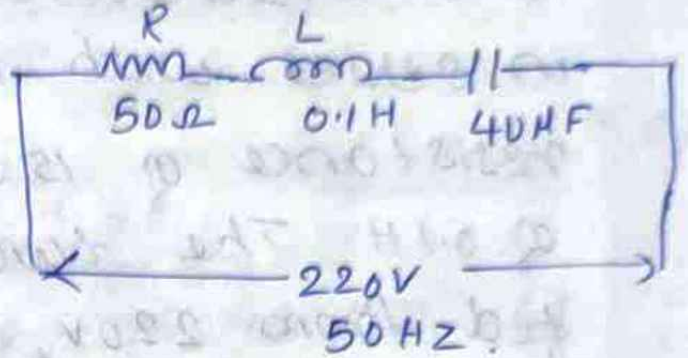
$$\text{avg. power, } P = VI \cos \phi = \underline{\underline{261.56 \text{ W}}}$$

$$\text{Reactive power, } Q = VI \sin \phi = 230 \times 7.23 \times \sin 80.95^\circ = \underline{\underline{1642.19 \text{ VAR}}}$$

$$\text{Apparent power, } S = VI = 230 \times 7.23 = \underline{\underline{1662.9 \text{ VA}}} \\ = \underline{\underline{1.662 \text{ kVA}}}$$

3. A resistor of 50Ω , an inductor of 0.1 H and a capacitor of $40 \mu\text{F}$ are connected in series and the combination is connected across 220 V , 50 Hz supply. calculate
- the circuit impedance

ii) resulting current iii) p.f iv) phase angle and v) power consumed by the



$$X_L = \omega L$$

$$= 2\pi f L$$

$$= 2\pi \times 50 \times 0.1$$

$$= 31.41 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 40 \times 10^{-6}}$$

$$= 79.57 \Omega$$

$$Z = R + j(X_L - X_C)$$

$$= 50 + j(31.41 - 79.57)$$

$$= 50 - j48.16$$

$$= 69.42 \angle -43.92^\circ$$

$$I = \frac{V}{Z} = \frac{220}{69.42 \angle -43.92^\circ} = 3.169 \angle 43.92^\circ$$

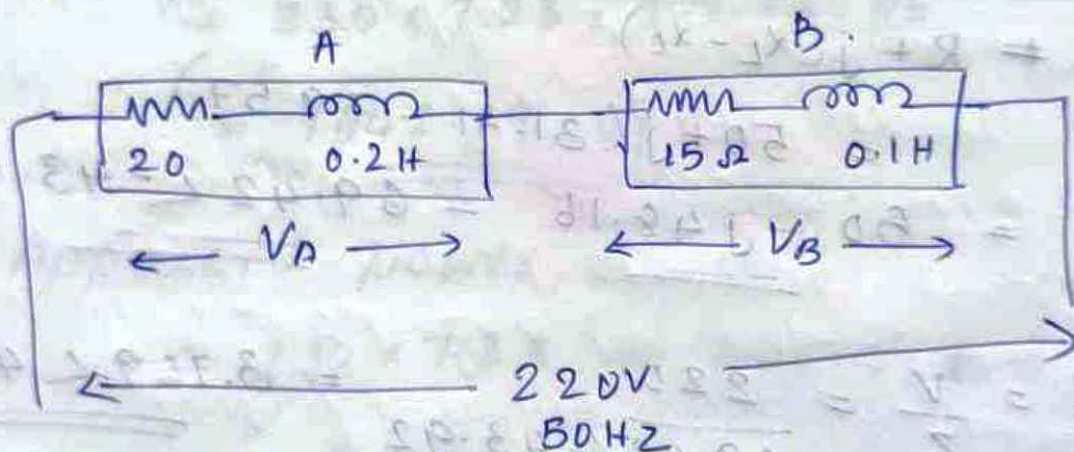
$$p.f = \cos 43.92^\circ = 0.720 \text{ leading}$$

$$\text{phase angle} = 43.92^\circ$$

$$P = VI \cos \phi = 220 \times 3.169 \times 0.72$$

$$= 501.96 \text{ W}$$

4. coil A having resistance of 20Ω and inductance of 0.2H is connected in series with another coil B having resistance of 15Ω and inductance of 0.1H . The two coils in series are fed from 220V , 50Hz , single phase power supply. Determine i) the voltage across each coil ii) power dissipated in each coil iii) pf of the whole ckt.



$$X_{LA} = 2\pi f L_A = 2\pi \times 50 \times 0.2$$

$$= 62.83\Omega$$

$$Z_A = 20 + j62.83 = 65.93 \angle 72.34^\circ$$

$$X_{LB} = 2\pi f L_B = 2\pi \times 50 \times 0.1$$

$$= \underline{\underline{31.41\Omega}}$$

$$Z_B = 15 + j31.41 = 34.81 \angle 64.47^\circ$$

$$Z = Z_A + Z_B$$

$$= 65.93 \angle 72.34^\circ \times 34.81 \angle 64.47^\circ$$

$$= \underline{100.53 \angle 69.62^\circ \Omega}$$

$$\underline{I} = \frac{V}{Z} = \frac{220}{100.53 \angle 69.62^\circ} = \underline{2.18 \angle -69.62^\circ}$$

$$V_A = \underline{I} Z_A = 2.18 \angle -69.62^\circ \times 65.93 \angle 72.34^\circ$$
$$= \underline{143.72 \angle 2.72^\circ}$$

$$V_B = \underline{I} Z_B = 2.18 \angle -69.62^\circ \times 34.81 \angle 64.47^\circ$$
$$= \underline{75.57 \angle -6.81^\circ}$$

power absorbed in coil A

$$= I^2 R_A = 2.18^2 \times 20$$

$$= \underline{95.048 \text{ W}}$$

power absorbed in coil B

$$= I^2 R_B = 2.18^2 \times 15$$

$$= \underline{71.286 \text{ W}}$$

$$p.f = \cos 69.62^\circ = 0.34 \text{ lagging}$$

6. A resistance of $10\ \Omega$, an inductance of $0.3\ \text{H}$ and a capacitance of $100\ \mu\text{F}$ are connected in series across $230\ \text{V}$, $50\ \text{Hz}$ single phase supply. Calculate i) the impedance of the ckt. ii) current through the ckt. iii) voltage across R , L and C , iv) power consumed by the ckt. v) p.f of the ckt.

$$X_L = 2\pi fL$$

$$= 2\pi \times 50 \times 0.3$$

$$= \underline{\underline{94.24\ \Omega}}$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$= \underline{\underline{31.83\ \Omega}}$$

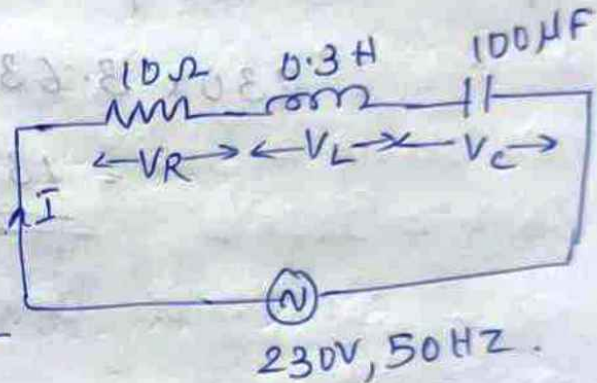
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = R + j(X_L - X_C) = 10 + j(94.24 - 31.83)$$

$$= 10 + j62.4 = \underline{\underline{63.2 \angle 80.89}}$$

$$I = \frac{V}{Z} = \frac{230 \angle 0}{63.2 \angle 80.89} = \underline{\underline{3.63 \angle -80.89}}$$

$$\text{P.f} = \cos 80.89 = 0.158 \text{ lagging}$$



$$V_R = IR = 3.639 \angle -80.89 \times 10 = 36.39 \angle -80.89$$

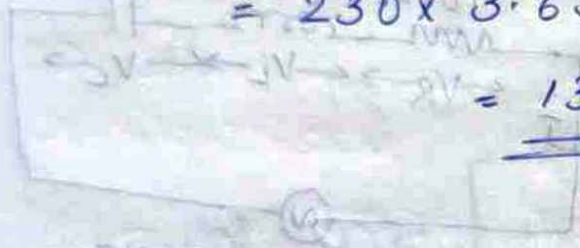
$$V_L = I X_L = 3.639 \angle -80.89 \times 94.24 \angle 90 = 342.95 \angle 9.11$$

$$V_C = I X_C = 3.639 \angle -80.89 \times 31.83 \angle -90 = 115.856 \angle -170.89$$

$$P = VI \cos \phi$$

$$= 230 \times 3.639 \times 0.158$$

$$= \underline{132.24 \text{ W}}$$



$$\frac{1}{Z} = \frac{1}{R + jX_L - jX_C} = \frac{1}{R + j(X_L - X_C)}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = I$$

$$(R + j(X_L - X_C))I = V \Rightarrow I = \frac{V}{R + j(X_L - X_C)}$$

$$P = VI \cos \phi = \frac{V^2 R}{R^2 + (X_L - X_C)^2}$$

$$P = \frac{V^2 R}{R^2 + (X_L - X_C)^2}$$

1. A balanced 3 phase load consist of 3 coils each of resistance 6Ω and inductive reactance of 8Ω . Determine the line current and power absorbed when the coils are
- star connected
 - delta connected
- across $400V$, 3 phase supply.

$$Z_{ph} = 6 + j8 \Omega$$

$$V_L = 400V$$

Star

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.9}{6 + j8} = 23.09 \angle -53.13^\circ A$$

$$I_L = I_{ph}$$

$$P = \sqrt{3} V_L I_L \cos \phi \quad \text{or} \quad 3 V_{ph} I_{ph} \cos \phi$$

$$= 3 \times 230.9 \times 23.09 \times \cos 53.13$$

$$= 9596.68 W = \underline{\underline{9.59 kW}}$$

Delta

$$V_{ph} = V_L = 400V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{6 + j8} = 40 \angle -53.13^\circ A$$

$$I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 40 \angle -53.13^\circ = \underline{\underline{69.28 \angle -53.13^\circ A}}$$

$$\begin{aligned}
 P &= 3 V_{ph} I_{ph} \cos \phi \\
 &= 3 \times 400 \times 40 \times \cos 53.13 \\
 &= \underline{\underline{28.8 \text{ kW}}}
 \end{aligned}$$

2. Three inductive coils, each with a resistance of 22Ω and an inductance of 0.05 H are connected in first star and then delta, to a 3 phase 415 V , 50 Hz supply. Calculate for both star and delta connections, i) phase current and line current and ii) total power absorbed.

$$Z = 22 + j10.10$$

$$\begin{aligned}
 X_L &= 2\pi fL = 2\pi \times 50 \times 0.05 \\
 &= \underline{\underline{15.7 \Omega}}
 \end{aligned}$$

$$Z = 22 + j15.7 \Omega$$

Star

$$V_L = 415 \text{ V}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = \underline{\underline{239.6 \text{ V}}}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{22 + j15.7} = \underline{\underline{8.87 \angle -35.52}}$$

$$I_L = I_{ph} = 8.87 \angle -35.52$$

$$P = 3 V_{ph} I_{ph} \cos \phi = 3 \times 239.6 \times 8.87 \cos 35$$

$$= \underline{\underline{5.183 \text{ kW}}}$$

Delta

$$V_L = V_{ph} = 415 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415}{22 + j15.7} = 15.37 \angle -35.53$$

$$I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 15.37 \angle -35.53$$

$$= \underline{\underline{26.62 \angle -35.53}}$$

$$P = 3 V_{ph} I_{ph} \cos \phi$$

$$= 3 \times 415 \times 26.62 \times \cos 35.53$$

$$= \underline{\underline{15.5 \text{ kW}}}$$

3. Three similar coils connected in star draw a total power of 15 kW at a power factor of 0.2 lagging from a 3 phase 400V, 50Hz power supply. Calculate the resistance and inductance of each coil.

$$P = 3 V_{ph} I_{ph} \cos \phi$$

$$1.5 \text{ kW} = 3 \times \frac{400}{\sqrt{3}} \times I_{ph} \cdot \cos \phi \times 0.2$$

$$\Rightarrow I_{ph} = 10.82 \text{ A} \angle -78.46$$

$$V_{ph} = 230.94$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230.94}{10.82 \angle -78.46} = 21.32$$

$$\cos^{-1} 0.2 = 78.46$$

$$= \underline{\underline{21.32 \angle +78.46}}$$

$$= \underline{\underline{4.26 - j 20.88}}$$

$$R = 4.26 \Omega$$

$$X_L = 20.88$$

$$L = \frac{X_L}{2\pi f} = \frac{20.88}{2\pi \times 50} = \underline{\underline{0.066 \text{ H}}}$$

4. A balanced delta connected 3 phase load is fed from a 3 phase 400V, 50 Hz power supply. The line current is 20 A and the total power absorbed by the load is 10 kW. calculate i) the impedance in each branch ii) the pf and iii) total power consumed if the same impedances are star connected.

$$V_L = V_{ph} = 400 \text{ V}$$

$$I_L = 20 \text{ A}$$

$$P = 10 \text{ kW}$$

$$I_{ph} = \frac{20}{\sqrt{3}} = 11.54 \text{ A}$$

$$10 \text{ kW} = \sqrt{3} V_L I_L \cos \phi$$

$$= \sqrt{3} \times 400 \times 20 \cos \phi$$

$$\cos \phi = \underline{0.72} \text{ lagging}, \phi = 43.94$$

$$Z = \frac{V_{ph}}{I_{ph}} = \frac{400}{11.54} \angle -43.94$$

$$= \underline{34.66} \angle 43.94$$

Star

$$V_L = 400, V_p = \frac{400}{\sqrt{3}} = 230.94$$

$$I_p = \frac{V_p}{Z_p} = \frac{230.94}{34.66 \angle 43.94} = \underline{6.66} \angle -43.94$$

$$P = 3 \times 230.94 \times 6.66 \times \cos 43.94$$

$$= \underline{3.3 \text{ kW}}$$

5. A balanced star connected load of impedance $6+j8\ \Omega$ is connected in a 3 phase 230 V, 50 Hz. Find line current and power absorbed by each phase.

$$Z = 6 + j8 = 10 \angle 53.13$$

$$V_L = 230\text{ V}, \quad f = 50\text{ Hz}$$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79\text{ V}$$

$$\bar{I}_p = \frac{V_p}{Z} = \frac{132.79}{10 \angle 53.13} = 13.27 \angle -53.13$$

$$I_p = \bar{I}_L = 13.27\text{ A}$$

$$\begin{aligned} P &= V_p I_p \cos \phi \times 3 \\ &= 132.79 \times 13.27 \times \cos 53.13 \\ &= \underline{\underline{1057.27\text{ W}}} \end{aligned}$$

6. A balanced three phase load has per phase impedance of $(30+j50)\ \Omega$. If the load is connected across 400 V, 3 phase supply. find i) phase current ii) line current and power supplied to load when it is connected in a) star b) delta.

$$Z_p = 30 + j50 = 58.31 \angle 59.036^\circ$$

Star

$$V_L = 400 \text{ V}$$

$$P_p = \frac{400}{\sqrt{3}} = 230.9 \text{ V}$$

$$i) \ I_p = \frac{V_p}{Z_p} = \frac{230.9}{58.31 \angle 59.036^\circ} = 3.96 \angle -59.036^\circ$$

$$I_L = I_p = 3.96 \angle -59.036^\circ$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

$$= \sqrt{3} \times 400 \times 3.96 \times \cos 59.036^\circ = 1411.81 \text{ W}$$

Delta

$$V_p = V_L = 400 \text{ V}$$

$$I_p = \frac{400}{58.31 \angle 59.036^\circ}$$

$$= 6.859 \angle -59.036^\circ$$

$$P = 3 V_p I_p \cos \phi$$

$$= 3 \times 6.859 \times 400 \times \cos 59.036$$

$$= \underline{\underline{4234.74 \text{ W}}}$$

$$I_L = \sqrt{3} I_p = \underline{\underline{11.86 \angle -59.03}}$$

7. The load to a 3 phase supply comprises of three similar coils connected in star. The line currents are 25 A and kVA and kW i/p are 20 and 11, resp. Find phase and line voltage. 2) the kVAR input 3) resistance and reactance of each coil.

$$I_L = 25 \text{ A}$$

$$I_p = 25 \text{ A}$$

$$\text{kVAR} = \sqrt{\text{kVA}^2 - \text{kW}^2} = \sqrt{20^2 - 11^2} = 16.7 \text{ kVAR}$$

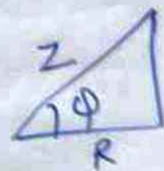
$$P = \sqrt{3} V_L I_L \cos \phi$$

$$S = 3 V_p I_p$$

$$V_p = \frac{20 \times 10^3}{3 \times 25} = \underline{\underline{266.66 \text{ V}}}$$

$$V_L = \sqrt{3} \times 266.66$$

$$= \underline{\underline{461.88 \text{ V}}}$$



$$p.f = \frac{kW}{kVA} = \frac{11}{20} = 0.55$$

$$Z_p = \frac{V_p}{I_p} = \frac{266.66}{25} = 10.66 \angle 56.63$$

$$= 5.86 + j8.9$$

$$R = Z \cos \phi = 10.66 \times 0.55 = 5.87 \Omega$$

$$X = \sqrt{10.66^2 - 5.87^2} = 8.92 \Omega$$

